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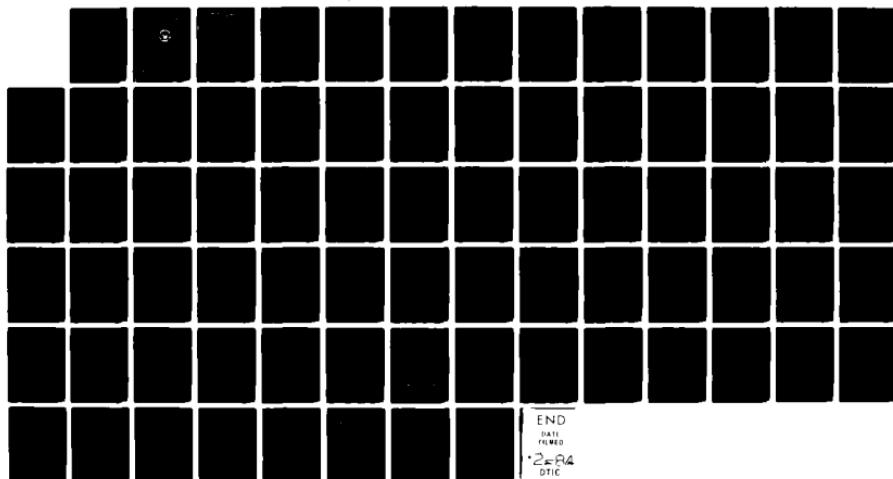
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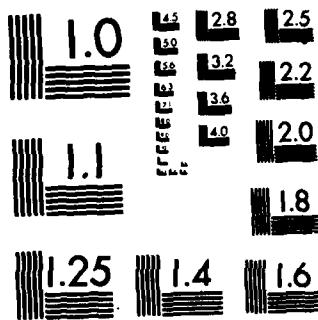
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THESIS

ANALYSIS OF INVENTORY MODELS
WITH BUDGET CONSTRAINT

by

Sung Jin, Kang

September 1983

Thesis Advisor: F.R. Richards

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are derived using the generalized Lagrange multiplier
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Sample data and output results are provided and
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a discussion and example is given of the use of the models
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Analysis of Inventory Models
with Budget Constraint

by

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Major, Republic of Korea Army
B.S., Republic of Korea Military Academy, 1974

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ABSTRACT

This thesis addresses the problem of determining the optimal number of spares for a multi-item inventory system with a procurement budget constraint. Various inventory models are considered with objective functions like time-weighted units short, units short, essentiality-weighted units short and pseudo-availability. Solution algorithms are derived using the generalized Lagrange multiplier approach and a marginal analysis approach.

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I. INTRODUCTION

In today's world, while all systems are becoming more and more sophisticated, the control and maintenance of inventories of these systems is a problem common to all enterprises and military services. In private and commercial concerns the effective control of inventories can result in decreased costs, increased sales and profits and consumer satisfaction. In the military proper management of inventories may contribute to increased availability and readiness, decreased inventory investment and system costs.

For each component of each weapon system two fundamental questions must be answered:

(1) When to replenish the inventory;

(2) How much to buy for the replenishment.

In order to answer these questions, many inventory models have been developed in the past 30 years. See for example, Hadley and Whitin [Ref. 1], Muckstadt [Ref. 2] and Eriksson [Ref. 3]. Most previous work solves a variety of cost minimization problems considering expected values of steady state variable costs associated with shortage cost, ordering cost and storage cost.

Such models may be appropriate for the commercial sector, but are not always appropriate in the military world.

In the commercial sector, the objective function of the inventory model is to maximize profit or minimize the average annual costs. Non-cost oriented objective functions frequently are used in the military inventory systems. For example, attempts are often made to maximize availability or fill rate, or minimize the number of backorders or expected time weighted stockouts, or minimize the probability of a stockout with a budget constraint.

Obviously costs are important in every inventory model. However, many real-world inventory problems are so complicated, one cannot represent accurately the real situation. Thus, some simplifications and approximations are used when constructing a mathematical model of any real world system. If this is not done, the results obtained by use of the model can easily lead to operating rules which are worse than those currently in use, worse than those which could be derived from simple heuristic intuitive considerations.

Many of the inventory problems are viewed as single period problems. For example, initial provisioning, allowance list determination and the fly-away kit problem are single period problems. These models are perhaps the simplest of the models in which demand is treated as a stochastic variable.

Reasonable objective functions in these models are to maximize performance subject to a constraint on the resources. Typical measures of performance might be availability, time-weighted units short, fill rate, the number of backorders, and mean supply response time.

This thesis considers various single period models which attempt to maximize performance subject to budget constraint.

Chapter II describes the general single period problem and introduces the method used in this thesis of solving those problems.

Chapters III and IV develop the time-weighted units short model and the availability model, and explain the solution procedure. Sample data runs for both models are provided.

Chapter V provides a comparison of models considered in the thesis and discusses some of the properties of each model, and Chapter VI discusses the use of models for purposes of determining the budget required.

Chapter VII summarizes the results of the research and concludes with some suggestions for additional research.

II. THE GENERAL PROBLEM

In this chapter, we consider the general single period model as a process for transforming resources into new distributions of inventory positions over the line items in the inventory.

The essential problems of control in a line item-inventory control system with multiple line items are:

(1) How much resources to commit at a point in time;

(2) How shall these resources be allocated to achieve system objectives.

In a typical continuous review inventory system, we can determine the optimal order quantity (Q) and reorder point (r) for a given item by minimizing the average annual variable costs. But in applying this theory to the real wo-ld inventory systems which consist of multiple line items, it is frequently the case that resulting minimum cos solutions are not feasible because of a budget limitation or some other constraint. Thus, in a constrained multi-item inventory system, the typical continuous-review policy is sometimes inappropriate. In the following section we discuss several objective functions to guide the line item inventory control system in determining how to allocate availabile procurement funds at a particular replenishment epoch.

A. GENERAL FORM OF OBJECTIVES WITH CONSTRAINTS

Consider the case in which an administrator, responsible for the replenishment decisions, determines replenishment of stocks of various line items on a periodic basis. Suppose that a fixed amount of procurement budget has been allocated to the replenishment epoch at hand and that a target number of reorder actions has been established as a working constraint for the allocation epoch. The administrator's task is to transform the available resources into replenishment orders for different items.

1. Measure of Effectiveness and Objective Function

Daeschner [Ref. 5] examined the constrained line-item allocation problems. He considered several possible objective functions which can be adapted to the case where unsatisfied demands are backordered and to the case where unsatisfied demands are lost sales. Let $\pi_j > 0$ be the penalty (reward) per unit for item j and let D_j be the demand for item j in a period. Let X_j be the inventory position for item j after ordering in a period. Let $D_j = d_j$. Then the number of sales for item j in the period is given by

$$d_i \quad \text{if} \quad d_j \leq X_j$$

$$X_j \quad \text{if} \quad d_j > X_j$$

The expected sales for item j is therefore

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$$\sum_{d_j=1}^{X_j} d_j \, p(D_j = d_j) + X_j \, p(D_j > X_j)$$

which is equivalent to

$$E(D_j) - \sum_{d_j=X_j+1}^{\infty} (d_j - X_j) p(D_j = d_j) \ .$$

We assume that the inventory system seeks to minimize the expected penalty incurred, or, equivalently, to maximize the expected penalty avoided. Mathematically, the objective is to maximize

$$Z(\underline{X}) = \sum_{j=1}^{N} \pi_j (E(D_j) - \sum_{d_j=X_j+1}^{\infty} (d_j - X_j) p(D_j = d_j))$$

Several interpretations and uses of the penalty coefficient π_j are possible. Four are illustrated in Table I below. Each reflects a formulation of system objectives which has been adopted or considered by the Navy Supply System. Daeschner [Ref. 5] also considered π_j as a linear combination of various coefficients in his line item allocation model.

There are many other types of objective functions which are currently used in the military. For example,

(1) Minimize units short in a given period

$$Z(\underline{X}) = \sum_{j=1}^{N} \sum_{d_j=X_j+1}^{\infty} (d_j - X_j) \, p(D_j = d_j)$$

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TABLE I

INTERPRETATIONS AND USES OF π_j

Penalty Coefficients	Objective
$\pi_j = c_j$	Maximize expected sales from stock.
$\pi_j = 1/\mu_j$	Maximize the expected requisitions filled (μ_j = average quantity of item j demanded per requisition).
$\pi_j = 1$	Maximize the expected number of units issued from stock.
$\pi_j = LT_j + TMNIS - TMISS$	Maximize expected customer waiting time per unit avoided by issue from stock, where LT_j is the lead time for item j, TMNIS is the calendar time anticipated to process a request and TMISS is the time to affect issue from stock of a demand, available item.

(2) Minimize time-weighted units short

$$Z(\underline{X}) = \sum_{j=1}^{N} TWUS_i(X_i) \cdot E_i$$

where:

$TWUS(X_i)$ = time weighted units short

E_i = essentiality

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(3) Maximize system availability

$$Z(\underline{X}) \; = \; \prod_{i=1}^{N} A_i(X_i)$$

where:

A_i = Availability for each item.

The objective functions (2) and (3) will be explained in Chapters III and IV.

   2. The Line-Item Allocation Model and Solution Procedure

In the previous sections, many kinds of objective functions are introduced. If we define them correctly, those objective functions can be solved by various techniques. It is evident that an actual inventory system with limited resources might be unable to carry out a prescribed inventory policy if either the amount of procurement funds available or the number of replenishment actions exceed the available resources. The problem is made more complicated by the fact that the objective functions are "non-linear" and the requirement that the X_j's must be integers. The problem is stated mathematically as

max $Z(\underline{s})$

(A1)   s.t. $\displaystyle\sum_{j=1}^{N} c_j s_j \; \leq \; B$

$\displaystyle\sum_{j=1}^{N} H(s_j) \; \leq \; R$

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$$\underline{s} \;=\; (s_1, s_2, \ldots, N): \quad \text{integers}$$

$$c_j \;=\; \text{the unit price of item } j$$

$$s_j \;=\; \text{the number of buys of item } j$$

$$H(s_j) \;=\; 1 \quad \text{if } s_j > 0$$
$$\qquad\;\; =\; 0 \quad \text{otherwise}$$

$$B \;=\; \text{the procurement budget limit at the reallocation epoch}$$

$$R \;=\; \text{the maximum number of individual procurement activities allowed in the present allocation.}$$

To solve the problem (A1), the generalized Lagrange multiplier (GLM) method of Everett [Ref. 4] can be used. Using this method, the problem can be reexpressed as

$$(A2) \qquad \max_{\underline{s}} \; L(\underline{S}, \underline{\lambda}) \;=\; Z(\underline{s}) \;-\; \lambda_1 \left(\left(\sum_{j=1}^{N} c_j S_j \right) - B \right)$$

$$- \lambda_2 \left(\left(\sum_{j=1}^{N} H(s_j) \right) - R \right)$$

$$\underline{S} \in s \text{ and } \lambda_1, \; \lambda_2 \geq 0 \text{ with optimal solution } \underline{S}^*(\underline{\lambda}).$$

Problem (A2) is the Lagrangian problem associated with (A1). Using Everett's theorem, one can determine a bound on the optimal solution, $Z(\underline{s}^*)$ to be

$$Z(\underline{s}^*) \;\leq\; Z(\underline{s}^*(\underline{\lambda})) \;-\; \lambda_1 (B(\underline{\lambda}) - B) \;-\; \lambda_2 (R(\underline{\lambda}) - R)$$

where

$$B(\underline{\lambda}) = \sum_{j=1}^{N} c_j S_j^*(\underline{\lambda})$$

$$R(\underline{\lambda}) = \sum_{j=1}^{N} H(S_j^*(\underline{\lambda}))$$

and $\underline{s}^*(\lambda)$ is the optimal solution vector for a given pair (λ_1, λ_2). In solving problem (A2), we can separate the N-variable optimization problem into N one-variable problems. Choosing trial values of λ_1 and λ_2 we maximize

(A3) $$L_j(s_j, \underline{\lambda}) = z_j(s_j) - \lambda_1 c_j s_j - \lambda_2 H(s_j) \ .$$

When considering the integer nature of decision variables, the optimal solutions for (A3) are determined by finding the values s_j^* such that

$$L_j(s_j^*+1, \underline{\lambda}) - L_j(s_j^*, \underline{\lambda}) \leq 0 \quad \text{and} \quad L_j(s_j^*, \underline{\lambda}) - L_j(s_j^*-1, \underline{\lambda}) > 0.$$

Thus s_j^* is the smallest value such that

$$\Delta L_j(s_j, \underline{\lambda}) = L_j(s_j+1, \underline{\lambda}) - L_j(s_j, \underline{\lambda}) \leq 0$$

In order to get an optimal solution, Daeshner [Ref. 5] used an interactive computer program, which evaluates the current

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optimal solutions with λ_1 and λ_2. Each time, the user can select a pair λ_1, λ_2 and objective function type to be considered. Then the user is provided with output which indicates the budget consumed, the number of stock replenishments generated, the achieved objective function value and a maximum attainable value for the objective function.

After examining the output, the user can modify the input parameters and continue or terminate the run. Decreasing the non-negative multiplier values tends to use more of the corresponding resources, increasing the values used, less. When the replenishment actions generated by a pair of values (λ_1, λ_2) exactly consume the available resources, B and R, the solution is optimal. Frequently exact equality may be impossible because of integer nature of the problem. Thus the solution obtained may not be optimal, but the difference is not likely to be significant.

B. AUTOMATING SEARCH ON THE LAGRANGE MULTIPLIER

The interactive search method cannot guarantee an optimal solution and it requires trial and error to get the approximate optimal solution. Consider the case in which there is only a single constraint, with the same type of objective functions. The mathematical program is then

$$\max_{\underline{s}} Z(\underline{s})$$

(B1)

$$\text{s.t.} \quad \sum_{j=1}^{N} c_i s_i \leq B$$

$$\underline{s} = (s_1, s_2, \ldots, s_N) = \text{integer number of buys}$$

$$B = \text{budget limit}$$

$$c_i = \text{price of item i.}$$

We can rewrite the above equation using a Lagrange multiplier, as:

$$(1) \qquad L(\underline{s}, \theta) = Z(\underline{s}) - \theta \left[\sum_{j=1}^{N} c_i s_i - B \right]$$

Then separate the equation.

$$(2) \qquad L(s_1, s_2, \ldots, s_N) = \sum_{j=1}^{N} (Z(s_i) - \theta c_i s_i) + \theta B$$

Equation (2) can be maximized by maximizing each sub-objective function. If $Z(s_i)$ is differentiable with respect to each s_i, the optimal solution is obtained by

$$\frac{\partial L}{\partial s_i} = \frac{dZ(s_i)}{ds_i} - \theta c_i, \qquad i = 1, 2, \ldots, N$$

Thus set $\dfrac{\partial L}{\partial s_i} = 0$ and get

$$\theta = \frac{dZ_i(s_i)}{ds_i} / C_i$$

where θ is such that $\sum_{j=1}^{N} c_i s_i = B$. Everett [Ref. 4] shows that θ can also be interpreted as a shadow price for the objective function: i.e., $\theta = \partial Z / \partial B$. Due to the integer

nature of s_i, it is often impossible to get an exact optimal solution. Difference equations must be used because the region in which the solution is desired consists of a set of discrete points. Therefore, let

$$(3) \qquad \Delta L_i(s_i,\theta) \;=\; L_i(s_i,\theta) - L_i(s_i-1,\theta)$$

$$= Z_i(s_i) - \theta c_i s_i - Z_i(s_i-1) + \theta c_i(s_i-1)$$

$$= \Delta Z_i(s_i) - c_i\theta$$

We know that Equation (3) is a concave function at the point $\underline{s} \geq 0$. The optimal solution must satisfy $\Delta L_i(s_i,\theta) \geq 0$ and $\Delta L_i(s_i+1,\theta) < 0$. Thus the optimal solutions are given by finding the largest s_i's such that

$$\Delta L_i(s_i,\theta) \;\geq\; 0$$

or equivalently

$$(4) \qquad \Delta Z_i(s_i) - c_i\theta \;\geq\; 0 \qquad i = 1,\ldots,N.$$

The Lagrangian multiplier θ can be found by the following search algorithm.

STEP 1. Find an initial upper bound θ_u. Let all s_i be assigned zero at the beginning and find the change of objective function per unit dollar as a result of increasing to one unit.

$$\theta_1 = \frac{\Delta Z_1(1)}{c_1}$$

$$\theta_2 = \frac{\Delta Z_2(1)}{c_2}$$

$$\vdots$$

$$\theta_n = \frac{\Delta Z_n(1)}{c_N}$$

where $\Delta Z_i(1) = Z_i(1) - Z_i(0)$.

Because of decreasing marginal returns or objective function values and because of the interpretation of θ, an upper bound on θ is given by: $\theta_u = \max[\theta_1, \theta_2, \ldots, \theta_n]$.

STEP 2. The initial θ_0 will be

$$\theta_0 = \frac{\theta_L + \theta_u}{2}$$

where $\theta_L = 0$.

Find for each i, the largest s_i so that

$$\frac{\Delta Z_i(s_i)}{c_i} \geq \theta_0$$

and evaluate the objective function and the budget required.

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STEP 3. If the budget used is greater than the given
budget, let

$$\theta_1 \;=\; \frac{(\theta_o + \theta_u)}{2}$$

otherwise

$$\theta_1 \;=\; \frac{\theta_o + \theta_L}{2}$$

Each time update the S vector, the objective
function values, the upper bound of the objective, and the
amount of budget consumed.

STEP 4. Stopping rule.

Stop when the used budget is equal to the given
budget or the difference between the current upper
bound and the objective function value is less
than some limit (ϵ). Otherwise go to step 2,
and continue until the above conditions are
satisfied.

A FORTRAN program for this algorithm is given in Appendix B.

C. MARGINAL ANALYSIS PROCESS

The theory of marginal analysis has been used in many
inventory models when resource constraints are active. In
an economic sense, $\Delta Z_i(s_i)/c_i$ can be interpreted as the
marginal increase in the objective function per dollar spent

achieved by adding one more unit of stock. It is reasonable for an inventory controller who has a scarce resource such as a procurement budget to buy an item which gives the maximum benefit per dollar spent.

By using a simple computerized algorithm, the line item allocation problem can be solved easily. The first step is to set all $s_i = 0$ and compute

$$\max_i \left[\frac{\Delta Z_1(s_1+1)}{c_1}, \frac{\Delta Z_2(s_2+1)}{c_2}, \ldots, \frac{\Delta Z_n(s_n+1)}{c_n}\right]$$

If the maximum is taken on for item j, set $s_j = 1$ and deduct the unit price for unit j from the budget. The second step is then to recompute ΔZ_j and then find

$$\max\left\{\max_{i \neq j}\left\{\frac{\Delta Z_i(s_i)}{c_i}\right\}, \frac{\Delta Z_j(s_j)}{c_j}\right\} .$$

The next unit is assigned to the index j where the maximum is taken on, etc. This is continued until adding an additional unit exceeds the budget constraint. It should be noted, however, that the method described does not insure optimality [Ref. 1]. Specifically the method may stop too soon. If the item i selected from the marginal analysis has a c_i value greater than the remaining budget, the procedure terminates even though some other item j may have a c_j value less than the remaining budget. An obvious improvement in this area could be the inclusion of a subroutine that would select

from the remaining items the best one from those having c_j's smaller than the remaining budget. A FORTRAN program for performing this marginal analysis is provided in Appendix C.

D. SAMPLE DATA RUNS OF UNITS SHORT MODEL

A weapon system consists of 10 components. The system manager wants ,to minimize the number of units short by supplying spare parts to support the weapon system. Suppose that the demand rate, lead time, price and essentiality code for each item i are known. The objective function can be expressed by

$$
(C1) \quad \text{minimize } Z(\underline{s}) = \sum_{i=1}^{10} \sum_{d_i=s_i+1}^{\infty} (d_i - s_i) p(D_i = d_i) E_i
$$

$$
\text{subject to} \quad \sum_{i=1}^{10} s_i c_i \leq B
$$

where:

$$E_i = \text{essentiality code}$$

$$B = \text{budget limit}$$

$$p(D_i = d_i) = \frac{e^{-\lambda_i T_i} (\lambda_i T_i)^{d_j}}{(d_i)!}$$

The approximate solution of (C1) can be obtained by the marginal analysis method. Table II shows the computational results for this system with known input data.

TABLE II

THE RESULT OF UNITS SHORT MODEL

No.	λ_i	Lead Time	Price ($)	Essen.	Alloca- tion	Units Short
1	1.0	1.0	10.2	1.0	5.0	0.0007
2	0.1	1.0	20.0	1.0	1.0	0.0048
3	3.0	1.0	100.0	1.0	2.0	1.2489
4	25.0	1.0	2.0	3.0	42.0	0.0013
5	1.0	1.0	5.0	1.0	5.0	0.0007
6	0.5	1.0	5.0	3.0	4.0	0.0002
7	10.0	1.0	1.0	1.0	21.0	0.0012
8	5.0	1.0	100.0	1.0	4.0	1.4368
9	1.0	1.0	50.0	1.0	3.0	0.0233
10	2.0	1.0	100.0	1.0	2.0	0.5413

Table II shows several properties of the units short model. First of all, more than one unit short in a year occurs in the high cost items (items 3 and 8). Second, low demands and low price items are allocated enough. Items 2, 5 and 6 are allocated more than five times their mean demand. Also this model tends to stock more of the high demands and low price items.

Finally, the essentiality weights cause greater allocations to be provided to those items with high essentiality than would be provided with equal weights.

Other results include:

Total objective value	3.26
Shadow price	0.001899
Budget limit	$1170
Budget left	$0.0

The shadow price is the last maximum value of $\frac{\Delta z(s_i) E_i}{c_i}$. It can be interpreted approximately as the amount of decrease in the objective function achieved by adding one more dollar.

III. TIME WEIGHTED UNITS SHORT MODEL

A. DESCRIPTION OF MODEL

In the previous chapter we have discussed various objective functions and solution methods for the single period inventory problem. In the military services, many measures of effectiveness have been used to indicate system performance. Among these measures are fill rate, availability, mean supply response time, the number of stockouts, and time-weighted units short. In this chapter we consider a model which minimizes time-weighted-units-short (TWUS).

Suppose that a weapon system consists of n components and the objective is to allocate a given budget for spare parts so as to minimize time-weighted-units short for the entire system. Assume that

(1) procurement lead time and repair lead time are known constants.

(2) demands for each installed unit have a known distribution.

(3) the total amount of procurement budget available to spend on all components is fixed.

(4) the objective is to minimize essentiality weighted TWUS. Mathematically, the model can be written as

$$\min_{\underline{s}} \sum_{i=1}^{n} TWUS_i(s_i) E_i \cdot \frac{1}{SLT}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} c_i s_i \leq B$$

where:

$TWUS_i(s_i)$ = time weighted units short when there are s_i units for item i

SLT = total sum of lead time demand $(\sum_{i=1}^{n} \lambda_i T_i)$

E_i = essentiality code for item i

B = budget limit in a given period

C_i = price of each item.

In the above problem, if the TWUS is properly defined, this model will be solved easily by using the methods explained in Chapter II.

B. POISSON DEMAND CASE

We shall now determine an exact expression for the $TWUS_i(s_i)$ for the case in which demands are Poisson distributed. Let the mean rate of demand be λ_i units per year and the lead time be a constant T_i. In addition to treating the demand variable as being discrete, the number of buys s_i also will be treated as a discrete variable. Thus if D_i is the lead time demand item j:

$$p(D_i = s_i) \;=\; \frac{e^{-\lambda_i T_i}(\lambda_i T_i)^{s_i}}{(s_i)!} \tag{1}$$

$$=\; p(s_i; \lambda_i T_i)$$

Let

$$\bar{p}(s_i) \;=\; \text{prob}(D_i \geq s_i) \;=\; \sum_{d_i = s_i}^{\infty} p(d_i; \lambda_i T_i) \tag{2}$$

If there are s_i units of stock for item i, Richards and McMasters [Ref. 8] show that the expected time-weighted units short in $(0, T_i)$ is given by

$$E[\text{TWUS}_i(s_i)] \;=\; \frac{T_i}{2}\{\bar{p}(s_i+1)\,[\lambda_i T_i - 2s_i + \frac{s_i(s_i+1)}{\lambda_i T_i}]$$

$$+\; p(s_i; \lambda_i T_i)(\lambda_i T_i - s_i)\} \tag{3}$$

For those cases where the expected lead time demand is large, the Poisson probabilities in (3) can be approximated by a normal distribution with mean $\lambda_i T_i$ and variance $\sigma_i^2 = \lambda_i T_i$. Let

$$\phi(x) \;=\; \frac{1}{\sqrt{2\pi}}\exp(-x^2/2)$$

be the standard normal probability density function and let $\Phi(x) = \int_x^{\infty} \phi(u)\,du$ be the complementary cumulative distribution

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function for the standard normal. Then expression (3) can
be rewritten in terms of the normal probability function as
follows:

$$E[TWUS_i(s_i)] = \frac{T_i}{2}\{\phi(\frac{s_i+1-\lambda_i T_i}{\sigma_i})\,[\lambda_i T_i - 2s_i + \frac{s_i(s_i+1)}{\lambda_i T_i}]$$

$$+ \frac{1}{\sigma_i}\,\phi(\frac{s_i-\lambda_i T_i}{\sigma_i})\,(\lambda_i T_i - s_i)\}\qquad(4)$$

This expression should be used in those cases in which $\lambda_i T_i$
is large. We have developed the expression for the expected
time-weighted units short in a period of length T_i when there
are s_i units of stock for item i.

In the next section we write the expression for the total
essentiality-weighted time weighted units short over all items,
and we provide a solution procedure for allocating the given
budget optimally.

C. SOLUTION PROCEDURE

The mathematical program for the time-weighted-units short
problem is:

$$(C1)\quad \min Z(\underline{s}) = \frac{1}{SLT}\sum_{i=1}^{N}\frac{E_i T_i}{2}\{\bar{p}(s_i+1)\,[\lambda_i T_i - 2s_i + \frac{s_i(s_i+1)}{\lambda_i T_i}]$$

$$+ p(s_i;\lambda_i T_i)(\lambda_i T_i - s_i)\}$$

$$\text{s.t.}\quad \sum_{i=1}^{N} c_i s_i \leq B$$

To solve this problem we can use the Lagrangian multiplier technique. Let

$$L(s_1, s_2, \ldots, s_n; \theta) = \sum_{i=1}^{N} Z_i(s_i) + \theta(B - \sum_{i=1}^{N} c_i s_i) \qquad (5)$$

Here Equation (5) is separable in the items, and minimization of the total objective function is accomplished by minimizing the individual functions $Z_i(s_i)$ subject to budget constraints. Consider a single item i. Let

$$\Delta L_i(s_i) = L_i(s_i-1) - L(s_i)$$

$$= Z_i(s_i-1) - Z_i(s_i) + \theta c_i s_i - \theta c_i(s_i-1)$$

$$= \Delta Z_i(s_i) + \theta c_i \qquad (6)$$

where

$$\Delta Z_i(s_i) = E_i[TWUS_i(s_i-1) - TWUS_i(s_i)] \qquad (7)$$

As shown earlier

$$TWUS(s-1) = \frac{T}{2}\{\overline{P}(s)[\lambda T - 2(s-1) + \frac{s(s-1)}{\lambda T}] + p(s-1; \lambda T)(\lambda T - s + 1)\}$$

$$= \frac{T}{2}\{\overline{P}(s)[\lambda T - 2s + \frac{s^2 - s}{\lambda T} + 2] + \frac{s}{\lambda T} p(s; \lambda T)(\lambda T - s + 1)\}$$

$$\text{TWUS}(s) = \frac{T}{2}\{\overline{P}(s+1)\,[\lambda T-2s+\frac{s(s+1)}{\lambda T} + p(s;\lambda T)\,(\lambda T-s)\}$$

$$= \frac{T}{2}\{\overline{P}(s)\,[\lambda T-2s+\frac{s(s+1)}{\lambda T}]-p(s;\lambda T)\,(\lambda T-2s+\frac{s(s+1)}{\lambda T})$$

$$+ p(s;\lambda T)\,(\lambda T-s)\}$$

so that

$$\text{TWUS}(s-1) - \text{TWUS}(s)$$

$$= \frac{T}{2}\{\overline{P}(s)\,[2 +\frac{s^2-s-s^2-s}{\lambda T}]+p(s;\lambda T)\,[s - \frac{s^2}{\lambda T} + \frac{s}{\lambda T} + \lambda T-2s$$

$$+ \frac{s^2+s}{\lambda T} - \lambda T + s]\}$$

$$= \frac{T}{2}[\overline{P}(s)\,[2 - \frac{2s}{\lambda T}] + \frac{2s}{\lambda T}\,p(s;\lambda T)]$$

$$= \overline{P}(s)\,[T -\frac{s}{\lambda}) + \frac{s}{\lambda}\,p(s;\lambda T) \qquad (8)$$

Substitute Equations (7) and (8) into (6). Then

$$\Delta L_i(s_i) = E_i[(T_i -\frac{s_i}{\lambda_i})\overline{P}(s_i;\lambda_i T_i) +\frac{s_i}{\lambda_i}\,p(s_i;\lambda_i T_i)]$$

$$+ \theta c_i \qquad (9)$$

The optimum solution s_i^* is the largest s_i such that

$$\Delta L_i(s_i) \geq 0$$

or equivalently,

$$\frac{E_i(Z_i(s_i-1)-Z_i(s_i))}{c_i} = \frac{E_i}{c_i}[(T_i - \frac{s_i}{\lambda_i})\bar{P}(s_i;\lambda_i T_i)$$

$$+ \frac{s_i}{\lambda_i} p(s_i)] \geq -\theta$$

The basic algorithm for solving this problem was explained
in the previous chapter. A computer program for searching
for θ is provided in Appendix B.

D. SAMPLE DATA RUNS

Consider a weapon system which consists of 10 components.
Suppose that the demand for each component is Poisson dis-
tributed with parameter λ_i and the lead time is known constant
T_i. Let the budget available for procurement be \$19224.
Table III shows the optimal allocations provided by the TWUS
model.

The allocation given when the demand distribution is
approximated by the normal distribution is also provided
in Table III for comparison. (For comparability, the variance
for the normal distribution is taken to be the same as the
mean). Comparing the results, we observe that the normal
case buys more of the high demands low cost items. There is
a small difference in the allocation for items 8 and 9 which
are more expensive than the others. For demand rates less
than 10, the usefulness of the approximation is questionable.

TABLE III

OPTIMAL ALLOCATION FOR TWUS MODEL

Item	λ_i	T_i	c_i (\$)	E_i	Poisson	Normal
1	10	1	10	1	17	20
2	100	1	20	1	113	120
3	15	1	80	1	19	21
4	20	1	2	1	32	36
5	50	1	5	1	65	71
6	80	1	30	1	90	96
7	20	1	1	1	35	37
8	15	1	200	1	17	15
9	75	1	100	1	77	74
10	10	1	75	1	14	16

The resulting values of the objective function for the optimal solutions are:

	Poisson Case	Normal Case
$Z(\underline{s}*)$	0.00094	0.0015
Shadow price ($\theta*$)	0.00015	0.00055
Budget limit	\$19224	\$19224
Budget left	0	0

The objective function value for the Poisson demand case is less than the normal demand case. The main reason for the difference is due to items 8 and 9.

IV. PSEUDO AVAILABILITY MODEL

A. DESCRIPTION OF MODEL

In the previous chapter, the TWUS objective function was introduced as a means for allocating a limited budget. Operational availability is a widely stated measure of the operational readiness of military forces and weapon systems. Thus, it is appropriate to consider stockage models with an availability objective as a means of allocating limited resources.

> The most direct and meaningful measure of the influence of peace time operating stocks on readiness is weapon system (or end item) availability. We use the terms availability, end-item availability, and weapon system availability interchangeably to mean the probability that an end item, such as a tank or an aircraft, selected at random, is not waiting for a component to be repaired or shipped to it. [Ref. 6]

Many authors have attempted to determine stockage levels for components by maximizing equipment operational availability, subject to a budget constraint. See, for example, Jee [Ref. 7]. Usually, the availability for component i is defined by the ratio

$$A_i = \frac{MTBF_i}{MTBF_i + MTTR_i + MSRT_i(s_i)}$$

where:

$MTBF_i$ = the Mean Time Between Failure of item i;

$$MTTR_i \quad = \quad \text{the Mean Time To Repair for item } i;$$

$$MSRT_i(s_i) \quad = \quad \text{the Mean Supply Response Time for item } i.$$

If the weapon system is assumed to consist of the n components all arranged in series, then the system availability is the product of the individual item availabilities. (This assumption means that the system will fail if any of the components fails.) With this assumption, the allocation problem, stated in terms of system availability is:

$$(P1) \qquad \max_{\underline{s}} \; \prod_{i=1}^{n} A_i(s_i)$$

$$\text{s.t.} \; \sum_{i=1}^{n} c_i s_i \leq B \qquad i = 1,2,\ldots,N$$

where:

$A_i(s_i) \quad = \quad$ the availability of item i having s_i units in stock;

$B \qquad\quad = \quad$ the budget limit;

$C_i \qquad\quad = \quad$ the price for each item i,

In the expression of $A_i(s_i)$, the term $MTBF_i$ is the reciprocal of the failure rate i, $MTTR_i$ is assumed to be independent of the decision variables and the available funds and $MSRT_i(s_i)$ can be expressed in terms of $TWUS(s_i)$ as

$$MSRT_i(s_i) \;=\; \frac{1}{\lambda_i T_i} \, TWUS_i(s_i).$$

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Thus, the main determination of availability from the point of view of the supply system is $MSRT_i(s_i)$. Many techniques for solving this model have been developed. In the next section we represent an algorithm for solving the availability model by using the marginal analysis method.

B. SOLUTION PROCEDURE

The model (P1) is not additive in the individual component availabilities but is converted into an additive function by transforming the objective function. Taking the natural log of the objective function, the model can be expressed in the following way.

$$(P2) \quad \max_{\underline{s}} \quad \sum_{i=1}^{n} \ln A_i(s_i)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} c_i s_i \leq B \qquad i = 1, 2, \ldots, N$$

Now the model (P2) is separable for all i and maximization of (P2) yields the same solution as maximization of (P1). The marginal analysis method selects an item which gives at each step the greatest increase in $\log(A_i(s_i))$ per dollar spent.

STEP 1. Start with zero units for all items.

STEP 2. Compute the increase in log availability per dollar spent as a result of purchasing one additional unit.

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$$\frac{\Delta \ln A_i(s_i)}{c_i} = \frac{1}{c_i}[\ln A_i(s_i) - \ln A_i(s_i)]$$

where $i = 1, 2, \ldots, N$

$$\ln A_i(s_i) = \ln[\frac{MTBF_i}{MTBF_i + MTTR_i + MSRT_i(s_i)}]$$

STEP 3. Select that item i corresponding to the maximum ratio.

$$\max_{\text{all } s_i} [\frac{\Delta \ln A_1(s_1)}{c_1}, \frac{\Delta \ln A_2(s_2)}{c_2}, \ldots, \frac{\Delta \ln A_n(s_n)}{c_n}]$$

STEP 4. Increase the number of units stocked for the item selected at step 3 by one additional unit if the unit price is less than the amount of budget remaining.

STEP 5. Update the S vector, the MSRT(s) expression and decrement the available budget. If the remaining budget is greater than the cost of the cheapest item, Go to Step 3. Otherwise, Stop.

In the following section, we will illustrate this procedure with a sample system. The computer program is provided in Appendix C.

C. A NUMERIC EXAMPLE FOR THE MSRT MODEL AND THE AVAILABILITY MODEL

In the expression for availability, the MTBF and MTTR terms are not functions of the number of spare parts. Therefore it is commonly believed that maximization of system availability is equivalent to minimization of mean supply response time. However, this is not the case, as shown below.

Suppose a weapon system consists of three components and the demands are Poisson distributed with parameters λ_1, λ_2 and λ_3, respectively. The lead time is a known constant and the components have essentiality codes E_i. The unit price and MTTR are known and the budget is limited to 20 dollars. This information is summarized in Table IV.

TABLE IV

INPUT DATA FOR EXAMPLE

ITEM	λ_i	c_i	$MTTR_i$	E_i	T_i
1	1	5	0.0274	1	1.0
2	0.1	5	0.0027	3	1.0
3	10	1	0.0054	1	1.0

To solve MSRT minimization problems, we first determine the MSRT's for all possible cases. These values are provided in Table V.

TABLE V

MSRT DATA FOR ALL FEASIBLE SOLUTIONS S_i

MSRT(s_i)	ITEM 1	ITEM 2	ITEM 3
MSRT(0)	0.9482	0.9837	0.6
MSRT(1)	0.3161	0.1967	0.5
MSRT(2)	0.0708	0.02	0.4099
MSRT(3)	0.0132	0.00227	0.3298
MSRT(4)	0.0021	0.0002	0.2596
MSRT(5)	0.0003		0.1992
MSRT(6)			0.1485
MSRT(7)			0.1072
MSRT(8)			0.0746
MSRT(9)			0.0500
MSRT(10)			0.0322
MSRT(11)			0.0199

Using the solution procedure described in the previous chapter we determine the optimal solution to be as shown in Table VI.

TABLE VI

THE ALLOCATION OF SPARE PARTS FOR MSRT MODEL

ITEM ALLOCATION	1 $\Delta z_1(s_1)$	2 $\Delta z_2(s_2)$	3 $\Delta z_3(s_3)$	USED BUDGET ($)
(0,0,0)	0.12462	0.47216	0.1	0
(0,1,0)	0.12462	0.1040	0.1	5
(1,1,0)	0.04905	0.1040	0.1	10
(1,2,0)	0.04905	0.01265	0.1	15
(1,2,1)	0.04905	0.01265	0.09	16
(1,2,2)	0.04905	0.01265	0.08012	17
(1,2,3)	0.04905	0.01265	0.07022	18
(1,2,4)	0.04905	0.01265	0.06039	19
(1,2,5)	0.04905	0.01265	0.0507	20

The optimal solution for MSRT model is (1,2,5). Repeating the analysis for the availability objective function we obtain the results provided in Table VII from the marginal analysis procedure.

$$\Delta z_i(s_i) = \frac{1}{c_i}[z_i(s_i+1) - z_i(s_i)]$$

$$= \frac{1}{c_i}[\ln A_i(s_i+1) - \ln A_i(s_i)]$$

TABLE VII

THE ALLOCATION OF SPARE PARTS FOR AVAILABILITY MODEL

ITEM ALLOCATION	1 $\Delta Z_1(s_1)$	2 $\Delta Z_2(s_2)$	3 $\Delta Z_3(s_3)$	USED BUDGET
(0,0,0)	0.07712	0.1294	0.1529	0
(0,0,1)	0.07712	0.1294	0.1611	1
(0,0,2)	0.07712	0.1294	0.1689	2
(0,0,3)	0.07712	0.1294	0.1759	3
(0,0,4)	0.07712	0.1294	0.1808	4
(0,0,5)	0.07712	0.1294	0.1821	5
(0,0,6)	0.07712	0.1294	0.1777	6
(0,0,7)	0.07712	0.1294	0.1660	7
(0,0,8)	0.07712	0.1294	0.1469	8
(0,0,9)	0.07712	0.1294	0.1217	9
(0,1,9)	0.07712	0.0447	0.1217	14
(0,1,10)	0.07712	0.0447	0.0938	15
(0,1,11)	0.07712	0.0447	0.06702	16
⋮		⋮		
(0,1,15)				20

The optimal solution for the availability model is (0,1,15). Comparing the results of the two models, we see that the availability model allocates more units to the high demand lower cost items than the MSRT model.

D. SAMPLE DATA RUNS

Suppose that a weapon system consists of 10 components and the demand of each component is Poisson distributed with parameter λ_i, and lead time T_i, mean time to repair $MTTR_i$ are known constants. In order to maximize the availability of spare parts with budget constraint, we can use the modified Availability model (P2) instead of (P1). By using the computer program in Appendix C, this problem can be solved. Table VIII provides the allocations of spare parts in the Availability model when the budget is 1170 dollars.

TABLE VIII

THE ALLOCATION OF SPARE PARTS FOR THE AVAILABILITY MODEL

ITEM	λ_i	T_i	c_i ($)	E_i	$MTTR_i$	ALLOCATION	$A_i(s_i)$
1	1.0	1.0	10.0	1.0	0.0137	4.0	0.986
2	0.1	1.0	20.0	1.0	0.0274	2.0	0.997
3	3.0	1.0	100.0	1.0	0.0137	3.0	0.821
4	25.0	1.0	2.0	3.0	0.0822	37.0	0.327
5	1.0	1.0	5.0	1.0	0.0274	5.0	0.973
6	0.5	1.0	5.0	3.0	0.0027	4.0	0.999
7	10.0	1.0	1.0	1.0	0.0054	21.0	0.949
8	5.0	1.0	100.0	1.0	0.0411	3.0	0.538
9	1.0	1.0	50.0	1.0	0.0082	3.0	0.987
10	2.0	1.0	100.0	1.0	0.1370	2.0	0.697

From the table, one can see that the availability of an item is greatly influenced by the MTTR term (see item 4). The availability for that item never exceeds 0.333 even if the MSRT is zero. We also observe that the availability model tends to stock the high demand low cost items.

The objective function for the optimal solution is given by:

Total obejctive value   0.08999

Shadow price   0.000261

Budget limit $1170

Budget left $0.0

A comparison of the above results with the allocation given in Table VIII shows that the total availability is relatively low even though most of the items have high availabilities. Also as mentioned above, when the MTTR data for an item is large relative to the MTBF, a high availability cannot be achieved.

V. COMPARISON OF MODELS

A. ANALYSIS FOR SAME DATA

In this chapter, we continue to consider the allocation
of spare parts to maximize the system performance in the
different allocation models. In this thesis, we have looked
at three models: the units short model, the time-weighted
units short model, and the availability model. Since each
model attempts to reduce stockouts as much as possible the
allocations generated by the models are strongly correlated.
This is especially true for the availability model and the
MSRT model since availability is a function of MSRT. However,
we saw earlier that the allocations from the models are not
necessarily the same.

Assume that a weapon system consists of 10 items, the
demands are Poisson distributed and $MTTR_i$, c_i, T_i, E_i are
known constants and a budget constraint of the weapon system
is \$1170. The optimal allocations for the three models are
shown in Table IX. As can be seen, the TWUS model is more
sensitive to the lead times than are the other two models
(see items 5, 6, and 7).

The units short model is more sensitive to the price of
the item than are the other two models. For item 9 the units
short model bought nothing, but the TWUS model and the availa-
bility model allocated 2 and 3 items respectively. All

TABLE IX

THE ALLOCATIONS OF SPARE PARTS FOR THE
THREE DIFFERENT MODELS

Item	λ_i (yr)	cost ($)	Ess.	T_i (yr)	MTTR (yr)	Units Short Model	TWUS Model	Avail. Model
1	1.0	10	1	1	0.0137	3	3	4
2	0.1	10	1	1	0.0137	2	1	2
3	15.0	3	1	1	0.0137	24	20	22
4	15.0	3	3	1	0.0274	26	23	24
5	3.0	10	1	0.5	0.0274	4	3	4
6	3.0	5	3	0.5	0.0274	6	4	6
7	10.0	50	1	0.2	0.0054	0	0	3
8	10.0	50	1	1	0.0411	8	7	2
9	2.0	50	1	1	0.0137	0	2	3
10	2.0	100	4	2	0.1370	5	5	5

three models are highly affected by the essentiality code.
This is illustrated by a comparison of items 9 and 10. Table
X presents the corresponding values of the three objective
functions.

TABLE X

THE COMPARISON OF OBJECTIVE VALUES FOR THREE MODELS

MODEL	OBJ FN UNITS SHORT	TWUS	AVAILABILITY
UNITS SHORT	0.1523	0.0458	0.0483
TWUS	0.1527	0.0353	0.0687
AVAILABILITY	0.1969	0.0775	0.0709

The above table was established by computing each objective function for the allocations determined by the three different procedures. Comparing the results of the three models, the TWUS model seems to do the best job considering all three objective functions. However, no general conclusions can be drawn about the preference of the TWUS model for other situations.

One needs to determine which objective function most closely matches a servicers' feeling about how operational readiness is affected by stockouts and delays in satisfying stockouts.

B. DISCUSSION OF SIMILARITIES

In the budget allocation problem there are many factors which affect the allocation such as demand, lead time, cost, time to repair and essentiality. The three models share similar properties. First of all, as can be seen in the above

example, all models tend to stock the cheap, high demand items in favor of expensive low demands items. This is because of the models attempts to get the biggest benefit per dollar spent. Potential benefit per additional unit increases with an items demand rate. Second, items having high essentiality code are given preference, as is the intent of essentiality assignment schemes. Essentiality weighting is one way to counter the preference given the high demand low cost item observed earlier. It is frequently the case that the most critical items are low demand expensive items. Without the essentiality weighting such items would be neglected by the type of models examined in this thesis.

VI.   USE OF THE MODELS FOR BUDGET DETERMINATION

A.   EFFECTIVENESS VS. BUDGET

The models that we have discussed have attempted to opti-
mize performance subject to a budget constraint.   We have
assumed that the budget was given.   There are many ways in
which budgets are determined.   However, budgeting people and
inventory managers alike often express the desire to have a
methodology that they can use to determine the amount of
money that should be provided.

In most cases the amount is determined historically by
giving an amount equal to what has been provided in the past
for similar systems or perhaps by giving a little more or
less based on judgement or financial constraints.   There is,
however, a strong interest brought about by Congressional
pressures to relate resources to readiness.   Congress wants
to know "how much money is needed to support our weapon sys-
tems at a specified level of performance."   In this chapter we
show how the models developed earlier in this thesis can be
used in just this manner.

Specifically, we show how the models that we have developed
can be modified easily to determine the minimum budget re-
quired to provide a specified level of logistics performance.

The models developed earlier can each be run for a range
of budget levels producing for each given budget an allocation

and a predicted overall level of performance. Figure 3 illustrates this for the case in which the performance measure is pseudo-availability. As expected, the curve shows that availability is a non-decreasing function of budget with decreasing marginal returns. This can be done also for the time-weighted units short model or any of the other models discussed in this thesis. In all cases we would obtain a similar display. Performance is a monotonic function of budget with decreasing marginal returns.

Figure 4 displays a similar result for the case in which the performance measure is MSRT. Each point on the curve represents an optimal level of performance for a given budget. For this example displayed, Figure 3, there is a little benefit to be gained by increasing the budget above $2500. However there is a dramatic increase in effectiveness obtained by increasing the budget from $1000 to $2000. This is precisely the sort of information needed to make intelligent budgeting decisions. Of course some decision maker must decide if the increase in effectiveness is worth the additional expenditure.

If a specific level of effectiveness is specified, one can graphically determine the amount of budget required by simply moving horizontally across the graph from the specified level of effectiveness until the curve is intersected and then down to the budget axis.
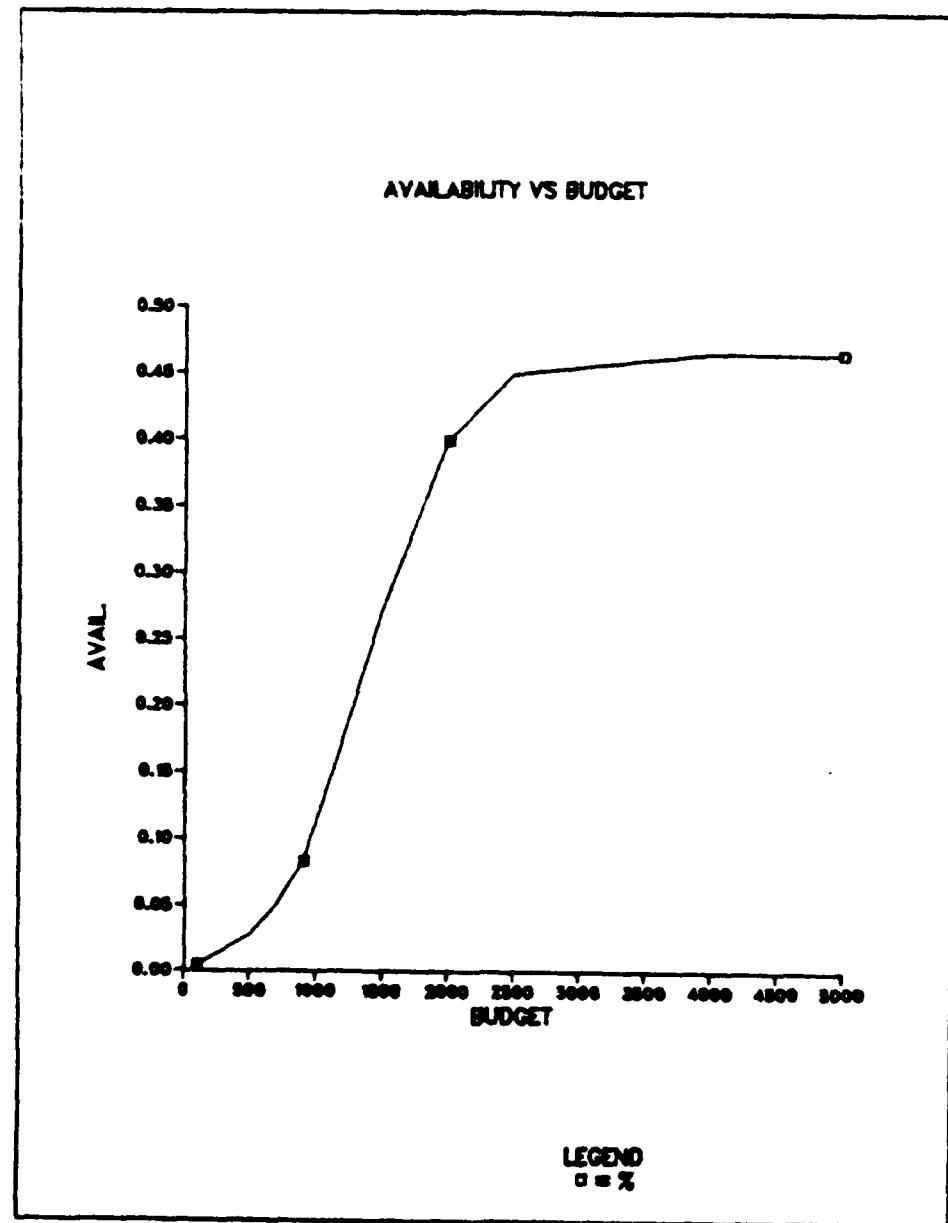
Figure 3.   Total Availability Vs. Budget Curve
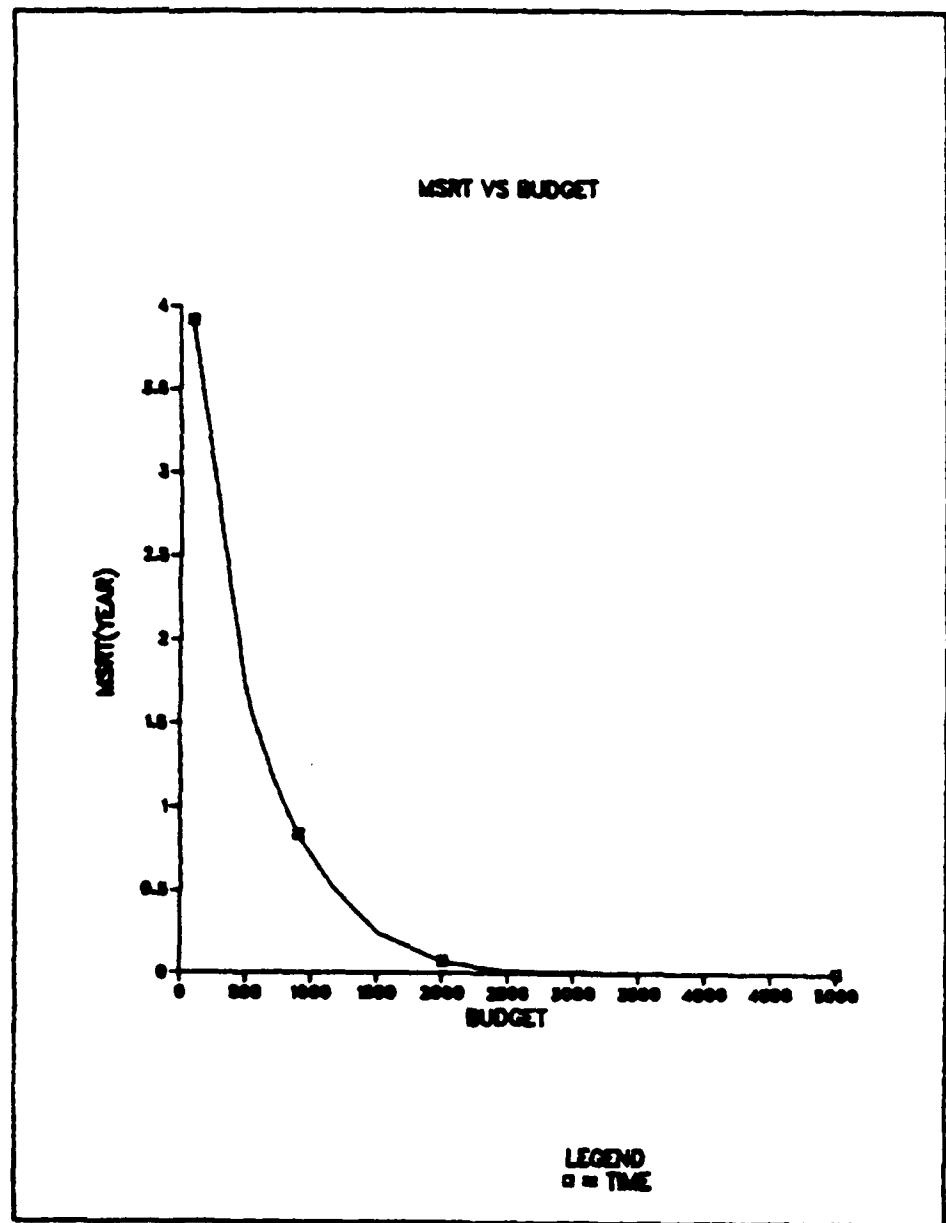
Figure 4.   MSRT Vs. Budget Curve

The next section determines analytically the minimum amount budget required by solving a companion problem to the problems discussed earlier in this thesis.

B. COMPANION PROBLEMS

In the previous chapters we have concentrated on the optimization of system effectiveness with a budget constraint. For many weapon systems such as air detection radars, missiles and nuclear delivery systems, the system performance is so important that the necessary budget will be provided to attain whatever performance is deemed necessary.

For such systems it is reasonable to restate the optimization problem to determine the minimum budget required to satisfy a specified level of performance. Consider, for example, the availability optimization problem and the companion problem:

$$\text{(D1)} \qquad \min \quad \sum_{i=1}^{n} c_i s_i$$

$$\text{s.t.} \quad \prod_{i=1}^{n} A_i \geq L \qquad i = 1,2,\ldots,n$$

where L is the minimum performance level for a weapon system.

For the MSRT model case the corresponding problem is:

$$\text{(D2)} \qquad \min \quad \sum c_i s_i$$

$$\text{s.t.} \quad \sum_{i=1}^{n} MSRT_i(s_i) \leq R \qquad i = 1,2,\ldots,n$$

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where R is the maximum allowable cumulative supply response
time for the weapon system.

Problems (D1) and (D2) can be solved using the same
methods explained in Chapters III and IV. In the above
models the budget is determined so that the system require-
ment for availability or main supply response time can be
achieved.

For problem (D2) the total cost is minimized when

$$\sum_{i=1}^{n} MSRT_i(s_i) = R .$$

Sometimes a minimum allowable supply response time is
required for each item. In such a case multiple constraints
could be specified. This is illustrated below:

(D3)   min $\sum_{i=1}^{n} c_i s_i$

$$MSRT_1(s_1) \leq R_1$$

$$MSRT_2(s_2) \leq R_2$$

$$\vdots$$

$$MSRT_n(s_n) \leq R_n$$

where R_i is the maximum allowable supply response time,
$i = 1,2,\ldots,N$.

To solve (D3), find the smallest s_i such that

$$\text{MSRT}_1(s_1) = R_1$$

$$\text{MSRT}_2(s_2) = R_2$$

$$\vdots$$

$$\text{MSRT}_n(s_n) = R_n$$

This problem is solved easily using the same procedures which we discussed.

So far we have discussed many different ways to apply the theoretical models to practical use of models for budget determination decisions. There is no unique method which gives us an optimal result. So the user of these models should choose one of possible methods so as to maximize the system performance or minimize the total cost.

VII. CONCLUSIONS

It is concluded that the various measures of effectiveness can be used in the budget constrained multi-item inventory system with stochastic demands. We have examined some of the more reasonable measures like minimization of units short, minimization of time-weighted units short and maximization of system availability. We have also looked at models which incorporate essentiality weights into each of the models.

In order to solve budget allocation stockage problems a feasible, efficient method of effecting line item inventory control is available using an adaptation of Everett's Generalized Lagrangian Multiplier method. Further, the use of a G.L.M. procedure provides valuable information for system managers as to relative effectiveness of additional procurement funds, versus additional transition processing capability. The final value of Lagrangian multipliers can be interpreted as the amount of improvement of the objective function per unit dollar spent.

The models discussed in this thesis are all more likely to stock cheap, high demand items than expensive, low demand items. Such is the nature of budget constrained optimization problems. If a system manager wishes to maintain enough stock for an item having low demand, high cost, his only alternative in our models is to assign a high essentiality

code for the item. The essentiality code has the effect of reducing the ratio C/E as opposed to C. In the solution procedure for each model, the assigned essentiality code directly affects the allocation for the item.

We have shown how the models can be used as a tool to determine the amount of budget. A simple graphical procedure allows a decision maker to determine the minimum budget required to search a specified level of performance. The optimization model is run several times to generate a plot of performance vs. budget. Each point of the curve represents the effectiveness for the optimal allocation of a given budget. A manager can, first, determine an appropriate system performance level and read from the curve the budget required to achieve the effectiveness.

Further analysis to improve these models may be possible. For instance, it would be useful to have an automatic search algorithm for the Lagrangian multipliers for a multiple constrained problem. It may also be possible to relax the assumptions for a constant lead time or a constant mean time to repair. These single period inventory may expand to time-dependent multi-item, multi-echelon, multi-indenture inventory systems.

COMPUTER PROGRAM FOR INTERACTIVE SEARCH

```
      DIMENSION KDEM(5,100),TMLD(100),IDUES(100),IONHD(100),
     1PRICE(100),SZRQN(100),INVNEW(100),PI(100),ZWT(100),TMSVD(
     2100),QUAL(100),KFREQ(5,100)
      COMMON CEMFCT(100),DEVFCT(100),IPOS(100)
      LOGICAL QUAL
      N=100
      NORDER = 0
      CSTBUD = 0.
      DATA IC1/' YES'/,IC2/ 'NO'/
      FLAG = 0.
      WRITE(6,137)
  137 FORMAT(' READ IN INITIAL VALUES.')
      READ(3,10) N,(ZWT(I),I=1,4),SHADW,SHADB,TMISS,TMNIS,J,R,WRKLD,
     1BUDGET
      DO 731 L=1,N
      READ(4,20) (KDEM(I,L),I=1,5),DEMFCT(L),PRICE(L),(KFREQ(I,L),I=1,5)
     1,DEVFCT(L),TMLD(L),IDUES(L),IONHD(L)
  731 FORMAT(14,6F5.2,3F6.2,F10.2)
   10 FORMAT(5I3,1X,2F5.2,1X,5I2,F6.2,5X,F2.1,7X,I2,I3)
   20 FORMAT(5I3,1X,2F6.2,1X,5I2,F7.2,5X,F3.1,7X,I2,I3)
   C1 DO 22 J=1,N
      DEM=0.
      DNUM=0.
      DO 23 K=1,5
      DDEM=KDEM(K,J)
      DFREQ=KFREQ(K,J)
   23 DNUM=DNUM+DFREQ
      DEM=DEM+DDEM
      SZRQN(J)=1.
      IF(DEM.LE.0.) GO TO 967
      SZRQN(J)=DNUM/DEM
  967 IF(SZRQN(J).LE.0.) SZRQN(J)=1.
      IPOS(J)=IDUES(J)+IONHD(J)
      TMSVD(J)=AMAX1(TMLD(J)+TMNIS-TMISS,0.)
   22 PI(J)=ZWT(1)*PRICE(J)+ZWT(2)*U+ZWT(3)*R/SZRQN(J)+ZWT(4)*TMSVD(J)
      ZSTARM = 0.
      ZS = 0.
      ZR = 0.
      ZU = 0.
      ZT = 0.
      ZMAXS = 0.
      ZSTAR = 0.
      ZMAXT = 0.
      ZMAXU = 0.
      FLAG=0.
      ZMAXR = 0.
  313 DO 100 J=1,N
      QTY1 = DEMFCT(J)
```

```
      IQTY2 = IPOS(J)                                               INV00490
      ZMAXS = ZMAXS + PRICE(J)*QTY1                                 INV00500
      VALDSC=CSCVAL((IQTY2+QTY1)                                    INV00510
      ZS=VALDSC*PRICE(J)+ZS                                         INV00520
      ZU=ZU+U*VALCSC                                                INV00530
      ZMAXU=ZMAXU+U*QTY1                                            INV00540
      ZMAXR=ZMAXR+R*QTY1/SZRQN(J)                                   INV00550
      ZR = ZR + R*VALCSC/SZRQN(J)                                   INV00560
      ZMAXT=ZMAXT + QTY1*TMSVD(J)                                   INV00570
      ZT = ZT + VALDSC*TMSVD(J)                                     INV00580
      ZSTAR = ZSTAR + VALDSC*PI(J)                                  INV00590
100   ZSTARM = ZSTARM +(PI(J))*QTY1                                 INV00600
      WRITE(6,40)(ZWT(J),I=1,4),ZSTAR,ZSTARM,ZS,ZU,ZR,ZT,ZMAXS,ZMAX INV00610
     1U,ZMAXR,ZMAXT                                                 INV00620
40    FORMAT(7X,'VALUE OF OBJECTIVE FUNCTION EVALUATED AT LEGACY FOR CHOINV00630
     1ICE OF WEIGHTS',//,14X,F6.4,2X,'FOR SALES',/,14X,F6.4,2X,'FOR UNITINV00640
     2S ISSUED',/,14X,F6.4,2X,'FOR REQNS ISSUED',/,14X,F6.4,2X,'FOR CUSTINV00650
     3OMER TIME SAVED',//,16X,'IS',2X,F10.2,'IF INFINITE RESOURINV00660
     4CES WERE AVAILABLE,THIS VALUE MIGHT BE INCREASED TO',F11.2,'//    INV00670
     56X,'VALUES OF LEGACY AND GREATEST POSSIBLE VALUE OF THE PURE OBJECINV00680
     6TIVE FUNCTIONS ARE',//,10X,'SALES',16X,'UNITS',16X,'REQNS',15X,'TIINV00690
     7ME',/,4F20.2,/4F20.2)                                         INV00700
      ZSTAR = 0.                                                    INV00710
      ZS = 0.                                                       INV00720
      ZU = 0.                                                       INV00730
      ZT = 0.                                                       INV00740
      ZR = 0.                                                       INV00750
      DO 200 J=1,N                                                  INV00760
312   ITRIAL = IPOS(J)+1                                            INV00770
      RUSS= SHADB * PRICE(J)/ PI(J)                                 INV00780
      D= DEMFCT(J)                                                  INV00790
      V = DEVFCT(J)                                                 INV00800
      BILL = ANDTRY(RUSS)                                           INV00810
      DEB = BILL*CEVFCT(J) + DEMFCT(J)                              INV00820
      APOS = IPOS(J)                                                INV00830
      IF(DEB.LT.APOS) DE B=APOS                                     INV00840
      MIKE = DEB +.5                                                INV00850
      ITRIAL = MIKE                                                 INV00863
201   VALDSC = DSCVAL(ITRIAL,J)                                     INV00870
      IBUYQ = ITRIAL - IPOS(J)                                      INV00880
      BUYQ = IBUYQ                                                  INV00890
      TSTBY =PI(J)* VALDSC - (SHADB * PRICE(J) * BUYQ) - SHADW      INV00900
      ICE = IPOS(J)                                                 INV00910
      ALICE = DSCVAL(ICE,J)                                         INV00920
      ISTNBY = PI(J)*ALICE                                          INV00930
      IF(TSTBY.GT.TSTNBY) GO TO 204                                 INV00940
      WINNER = ALICE                                                INV00950
      INVNEW(J) = IPOS(J)                                           INV00960
```

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```
199  ZSTAR = ZSTAR + PI(J)*WINNER                              INV00970
     ZS = ZS + WINNER*PRICE(J)                                 INV00980
     ZU = ZU + WINNER * U                                      INV00990
     ZR = ZR + WINNER * R / SZRQN(J)                           INV01000
     ZT = ZT + WINNER * TMSVD(J)                               INV01010
     GO TO 200                                                 INV01020
204  WINNER = VALDSC                                           INV01030
     INVNEW(J)=ITRIAL                                          INV01040
     NORDER = NORDER + 1                                       INV01050
     CSTBUD =CSTBUD + BUYQ * PRICE(J)                          INV01060
     GO TO 199                                                 INV01070
200  CONTINUE                                                  INV01080
     UBOUND = ZSTAR - (CSTBUD - BUDGET)*SHADB -(NORDER- WRKLD)*SHADW  INV01090
     WRITE(6,300) BUDGET, WRKLD                                INV01100
300  FORMAT(' ,F10.2,10X,'WORKLOAD LIMIT IS',F5.0)             INV01110
     WRITE(6,302) CSTBUD, NORDER, UBOUND                       INV01120
302  FORMAT(5X,'COST IS',F10.2,5X,'NORDER =',I4,5X,'UBOUND =',F10.2 )  INV01130
     WRITE(6,329) ZSTAR, ZS, ZU, ZR, ZT                        INV01140
329  FORMAT(5X,'ZSTAR =',F8.2,' ZS =',F8.2,'  ZU =',F8.2,'  ZR =',F8.2,'  ZR  INV01150
    1=',F8.2,' ZT =',F8.2)                                     INV01160
     WRITE(6,303)                                              INV01170
303  FORMAT(3X,'  TRY AGAIN?')                                 INV01180
     READ(5,304) IANS                                          INV01190
304  FORMAT(A3)                                                INV01200
     IF(IANS.EQ.IC2) GO TO 400                                 INV01210
     ZSTAR = 0.                                                INV01220
     ZU = 0.                                                   INV01230
     CSTBUD = 0.                                               INV01240
     ZS = 0.                                                   INV01250
     NORDER = 0.                                               INV01260
     ZR = 0.                                                   INV01270
     ZT = 0.                                                   INV01280
     ZMAXS = 0.                                                INV01290
     ZMAXT = 0.                                                INV01300
     ZMAXR = 0.                                                INV01310
     ZMAXU = 0.                                                INV01320
     FLAG = 0.                                                 INV01330
     ZSTARM = 0.                                               INV01340
     WRITE(6,305)                                              INV01350
305  FORMAT(3X,'  SAME  WEIGHTS?')                             INV01360
     READ(5,304) IANS                                          INV01370
     IF(IANS.EQ.IC1) GO TO 350                                 INV01380
     FLAG=1.0                                                  INV01390
310  WRITE(6,307)   INPUT ZS,ZU,ZR,ZT VIA 4F5.3'               INV01400
307  FORMAT(3X,'  INPUT ZS,ZU,ZR,ZT VIA 4F5.3')                INV01410
     READ(5,306) ZWT(1),ZWT(2),ZWT(3),ZWT(4)                   INV01420
306  FORMAT(4F5.3)                                             INV01430
     WRITE(6,308)(ZWT(I),I=1,4)                                INV01440
```

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```
308 FORMAT(3X,' WEIGHTS ARE',3X,4(F6.3,2X))                    INVO1450
309 FORMAT(3X,' CORRECT?')                                     INVO1460
    WRITE(6,309)                                               INVO1470
    READ(5,304) IANS                                           INVO1480
    IF(IANS.EQ.IC2) GO TO 310                                  INVO1490
    DO 800 J=1,N                                               INVO1500
800 PI(J)=ZWT(1)*PRICE(J)+ZWT(2)*U+ZWT(3)*R/SZRQN(J)+ZWT(4)*TMSVD(J) INVO1510
350 WRITE(6,363) SHADB, SHADW                                  INVO1520
363 FORMAT(3X,'SHADB =',F10.3,5X,'SHADW =',F10.3,/,'  SAME MULTIPLIER INVO1530
    1S?')                                                      INVO1540
    READ(5,304) IANS                                           INVO1550
    IF(IANS.EQ.IC1) GO TO 312                                  INVO1560
    WRITE(6,315)                                               INVO1570
315 FORMAT(3X,' INPUT SHADB AND SHADW VIA 2F10.3')             INVO1580
316 FORMAT(2F10.3)                                             INVO1590
    READ(5,316) SHADB, SHADW                                   INVO1600
    GO TO 312                                                  INVO1610
400 WRITE(6,401)                                               INVO1620
401 FORMAT(3X,' LOOK AT LINE ITEMS?')                          INVO1630
    READ(5,304) IANS                                           INVO1640
    IF(IANS.EQ.IC2) GO TO 500                                  INVO1650
    WRITE(6,981)                                               INVO1660
981 FORMAT(' HOW MANY?',/,'  XXX')                             INVO1670
982 READ(5,982) NUMI                                           INVO1680
982 FORMAT(I3)                                                 INVO1690
    WRITE(6,403)                                               INVO1700
403 FORMAT(' ITEM   LEGACY  NEW QTY')                          INVO1710
    DO 402 J=1,NUMI                                            INVO1720
    IF(IPOS(J).EQ.INVNEW(J)) GO TO 402                         INVO1730
    WRITE(6,404) J, IPOS(J), INVNEW(J)                         INVO1740
404 FORMAT(2X,I4,3X,I5,3X,I5)                                  INVO1750
402 CONTINUE                                                   INVO1760
    WRITE(6,403)                                               INVO1770
    DO 405 J=1,N                                               INVO1780
405 WRITE(6,404) J, IPOS(J),INVNEW(J)                          INVO1790
500 CONTINUE                                                   INVO1800
    STOP                                                       INVO1810
    END                                                        INVO1820
    FUNCTION DSCVAL(I,J)                                       INVO1830
    COMMON CEMFCT(100), DEVFCT(100), IPOS(100)                 INVO1840
    K=1                                                        INVO1850
    DSCVAL = 0.                                                INVO1860
    IF(I.LE.J) GO TO 182                                       INVO1870
    D = DEMFCT(J)                                              INVO1880
    V = DEVFCT(J)                                              INVO1890
    DSCVAL=DSCVAL + DLTAZJ(K,D,V)                              INVO1900
    IF(K.GE.I) GO TO 182                                       INVO1910
180 K=K+1                                                      INVO1920
```

```
182   GO TO 180
      RETURN
      END
      FUNCTION DLTAZJ(I,D,V)
      COMMON DEMF(T(100),DEVFCT(100),IPOS(100)
      SIGMA = 1.25*V
      AMIN = D*E
      IF(SIGMA.LT.AMIN) SIGMA = AMIN
      IF(SIGMA.LT.1.) SIGMA = 1.
      AI = (AI - D)/SIGMA
      X = (AI - D)/SIGMA
      AX = ABS(X)
      T = 1.0/(1.0 + .2316419*AX)
      D = 0.3989423 * EXP(-X*X/2.0)
      DLTAZJ=1.0 - D*T*(((1.330274*T - 1.821256)*T + 1.781478)*T
     1 - 0.356563)*T + 0.3193815)
      DLTAZJ=1,2:2   C - DLTAZJ
      IF(X) 1,2,2
    1 DLTAZJ=1.0 - DLTAZJ
    2 RETURN
      END
      FUNCTION ANCTRY(APROB)
      IF(APROB.GT..995)APROB=.995
      IF(APROB.LT..005)APROB=.005
    7 D = APROB
      IF(D - 0.5) 9,9,8
    8 D = 1.0 -D
    9 T2 = ALCG(1.0 /(D*D))
      T = SQRT((2.(2.515517+0.802853*T+0.010328*T2)/(1.0+1.432788*
     1T+0.189269*T2 + 0.001308*T*T2)
      IF(APROB - .5) 10,10,11
   10 X = -X
   11 D = 0.3989423*EXP(-X*X/2.0)
      X =-X
   12 ANDTRY = X
      RETURN
      END
```

COMPUTER PROGRAM FOR AUTOMATING SEARCH

```
                                        TH200010
                                        TH200020
                                        TH200030
                                        TH200040
                                        TH200050
                                        TH200060
                                        TH200070
                                        TH200080
                                        TH200090
                                        TH200100
                                        TH200110
                                        TH200120
                                        TH200130
                                        TH200140
                                        TH200150
                                        TH200160
                                        TH200170
                                        TH200180
                                        TH200190
                                        TH200200
                                        TH200210
                                        TH200220
                                        TH200230
                                        TH200240
                                        TH200250
                                        TH200260
                                        TH200270
                                        TH200280
                                        TH200290
                                        TH200300
                                        TH200310
                                        TH200320
                                        TH200330
                                        TH200340
                                        TH200350
                                        TH200360
                                        TH200370
                                        TH200380
                                        TH200390
                                        TH200400
                                        TH200410
                                        TH200420
                                        TH200430
                                        TH200440
                                        TH200450
                                        TH200460
                                        TH200470
                                        TH200480
```

```
C     VARIABLE DEFINITION
C
C     TMLD   : LEAD TIME (100)
C     TMEAN  : MEAN DEMAND (100)
C     TMLDM  : LEAD TIME DEMAND (100)
C     PRICE  : PRICE (100)
C     DELZ   : (Z(I)-Z(O))/PRICE(I) (100)
C     S      : THE NUMBER OF ITEM REQUIRED (100)
C     PBAR   : P(S) COMPLEMENTRY CUMULATIVE DISTRIBUTION (100)
C     ZSTAR  : THE VALUE OF OBJECTIVE FUNTION
C     CSTBUD : USED BUDGET
C     BUDGET : GIVEN BUDGET
C     UB     : UPPER BOUND
C     CUB    : CURRENT UPPER BOUND
C     THETA  : LAGLANGIAN MULTIPLIER
C     MTBF   : MEAN TIME BETWEEN FAILURE
C     MSRT   : MEAN SUPPLY RESPONSE TIME
C     MTTR   : MEAN TIME TO REPAIR
C     ESS    : ESSENTIALITY CODE
C
C=====================================================================
      DIMENSION DELZ(100),S(100),PBAR(100),MTTK(100),ESS(100),AAV(100),
     /TMLD(100),TMEAN(100),TMLDM(100),PRICE(100),MTBF(100),MSRT(100)
      REAL*4 DELZ,S,PBAR,ZSTAR,CSTBUD,UB,CUB,THETA,THETA1,THETAL,
     1Z1,SS,SAN,SS,OLD,X1,X2,X3,X4,X5,X6,X7,X8,CCDF,PO,ZS1,ZS2,
     1Z2,Z3,Z0,MTBF,MTTR,ESS,MSR,AAV
      INTEGER*4 I,J,K,N,KK
C=====================================================================
C     INPUT DATA
C=====================================================================
      N=10
      BUDGET=19224.0
      DO 11 I=1,N
      READ(5,12) TMEAN(I),TMLD(I),PRICE(I),MTTR(I),ESS(I)
      MTBF(I)=1.0/TMEAN(I)
   11 CONTINUE
   12 FORMAT(5F10.5)
C=====================================================================
C     FIND Z(1)-Z(0) ARRAY
C=====================================================================
      DO 13 I=1,N
      TMLDM(I)=TMEAN(I)*TMLD(I)
      CALL SUB(TMLDM(I),1.0,CCDF,CDF,PO)
      X1=CCDF
      X3=PO
```

```fortran
      CALL SUB (TMLDM(I),0.0,CCDF,CDF,PO)                               TH200490
      X2=CCDF                                                           TH200500
      Z1=(TMLC(I)-(1.0/TMLDM(I)))*X2                                    TH200510
      Z1=Z1+(1.0/TMLDM(I))*X3                                           TH200520
      DELZ(I)=Z1*ESS(I)/PRICE(I)                                        TH200530
   13 CONTINUE                                                          TH200540
C                                                                       TH200550
C     FIND UPPER VALUE OF THETA                                         TH200560
C                                                                       TH200570
C                                                                       TH200580
      THETAM=DELZ(1)                                                    TH200590
      DO 14 K=2,N                                                       TH200600
      IF(THETAM.GT.DELZ(K)) GO TO 14                                    TH200610
      THETAM=DELZ(K)                                                    TH200620
   14 CONTINUE                                                          TH200630
C                                                                       TH200640
      THETAL=0.0                                                        TH200650
      THETA=THETAM/2.0                                                  TH200660
      OLD=20.                                                           TH200670
      WRITE(6,906) THETAM                                               TH200680
  906 FORMAT(1X,' THE MAXIMUM VALUE OF THETA = ',F10.2)                 TH200690
C                                                                       TH200700
C     FIND S(1),S(2), , , , S(N)                                        TH200710
C                                                                       TH200720
  800 CONTINUE                                                          TH200730
      SUM=0.0                                                           TH200740
      DO 901 J=1,N                                                      TH200750
      SSS=1.0                                                           TH200760
      S2=SSS-1.C                                                        TH200770
      S3=SSS+2.C                                                        TH200780
      CALL SUB (TMLDM(J),S1,CCDF,CDF,PO)                                TH200790
      X1=CCDF                                                           TH200800
      CALL SUB (TMLDM(J),SSS,CCDF,CDF,PO)                               TH200810
      X2=CCDF                                                           TH200820
      X5=PO                                                             TH200830
      ZS1=(TMLC(J)-(SSS/TMLDM(J))*X1)                                   TH200840
      ZS1=(ZS1+(SSS/TMLDM(J))*X5)/PRICE(J)                              TH200850
      ZS1=ESS(J)*ZS1                                                    TH200860
      IF(X2.LE.0.00001) GO TO 902                                       TH200870
      IF(ZS1.LE.(THETA)) GO TO 902                                      TH200880
      SSS=SSS+1.0                                                       TH200890
      GO TO 903                                                         TH200900
  902 S(J)=SSS-1.0                                                      TH200910
      SUM=SUM+S(J)*PRICE(J)                                             TH200920
  903                                                                   TH200930
  902                                                                   TH200940
                                                                        TH200950
                                                                        TH200960
```

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```
901 CONTINUE
    WRITE(6,905)(S(I),I=1,10),THETA,SUM
905 FORMAT(1X,'S=',10F5.1,' THETA=',F9.5,' CST=',F9.1)
C =============================================================
C       COMPUTE Z(S) BY USING S(1),S(2),...,S(N)
C =============================================================
    UB=99999.0
    ZSTAR=0.0
    ZSTARL=0.0
    CSTBUD=0.0
    SLT=0.0
    DO 15 J=1,N
    SLT=SLT+TMLDM(J)
    CALL SUB (TMEAN(J),S(J)-1.0,CCDF,CDF,PO)
    X5=CCDF
    CALL SUB (TMLDM(J),S(J),CCDF,CDF,PO)
    X6=CCDF
    CALL SUB (TMLDM(J),S(J)+1.0,CCDF,CDF,PO)
    X7=CCDF
    CALL SUB (TMLDM(J),S(J)+2.0,CCDF,CDF,PO)
    X8=CCDF
    PBAR(J)=X6
    Z1=TMLDM(J)*X6
    Z2=2.0*S(J)*X7
    Z3=S(J)*(S(J)+1.0)*X8/TMLDM(J)
    ZO=0.5*TMLD(J)*(Z1-Z2+Z3)
    MSRT(J)=Z/TMLDM(J)
    AAV(J)=MTBF(J)/(MTBF(J)+MTTR(J)+MSRT(J))
    ZSTAR=ZSTAR+ZO*ESS(J)
    CSTBUD=CSTBUD+S(J)*PRICE(J)
15  CONTINUE
    ZSTAR=ZSTAR/SLT
    CUB=ZSTAR-THETA*(CSTBUD-BUDGET)
C =============================================================
C       SEARCH OPTIMAL THETA
C =============================================================
    IF ((CUB-ZSTAR)/CUB.LE.0.00001.OR.(CSTBUD-BUDGET).LT.1.0) GO TO
   /99
    IF (ABS(THETA-OLD).LE.0.00001.AND.(CSTBUD.LE.BUDGET)) GO TO 99
    IF (ABS((CUB-UB)/CUB).LE.0.001) GO TO 99
    IF ((BUDGET-CSTBUD).LE.1.).AND.((CSTBUD.GT.BUDGET)) GO TO 99
    IF (CUB.LE.UB) UB=CUB
    IF (CSTBUD.GT.BUDGET) GO TO 21
    THETAM=THETA
    THETA=(THETAL+THETAM)/2.0
    OLD=THETAM
    GO TO 800
```

TH200970
TH200980
TH200990
TH201000
TH201010
TH201020
TH201030
TH201040
TH201050
TH201060
TH201070
TH201080
TH201090
TH201100
TH201110
TH201120
TH201130
TH201140
TH201150
TH201160
TH201170
TH201180
TH201190
TH201200
TH201210
TH201220
TH201230
TH201240
TH201250
TH201260
TH201270
TH201280
TH201290
TH201300
TH201310
TH201320
TH201330
TH201340
TH201350
TH201360
TH201370
TH201380
TH201390
TH201400
TH201410
TH201420
TH201430
TH201440

```
21      THETAL=THETA
        QLD=THETA
        THETA=(THETAL+THETAM)/2.0
        GO TO 80C

C       ===============================================
C       OUTPUT PRINT
C       ===============================================

99      WRITE(6,100) THETA, ZSTAR,CUB,BUDGET,CSTBUD
        WRITE(6,101)
        WRITE(6,102)
        DO 200 I=1,A
        WRITE(6,103) I,PRICE(I),TMEAN(I),S(I),PBAR(I),AAV(I),MSRT(I)
200     CONTINUE
100     FORMAT(1H1,'//20X,' ***** THE RESULT OF SEARCH *****'///)
101     FORMAT(10X,'THETA      = ',F10.4/
       10X,'ZSTAR      = ',F10.4/
       10X,'CURRENT UPPER BOUND = ',F10.3/
       10X,'BUDGET LIMIT = ',F10.3/
       10X,'USED BUDGET  = ',F10.3//)
102     FORMAT(6X,'NO.',5X,'IM DEMAND ',5X,' BUY S ',5X,
       'PBAR (S)',5X,' AVIL (S)',5X,'MSRT(S)'//)
103     FORMAT(6X,I4,7X,F9.2,5X,F8.2,5X,F8.3,5X,F8.3,5X,F9.5)
        STOP
        END
        SUBROUTINE SUB(TT,SS,CCDF,CDF,PO)
        REAL*4 TT,SA,SS,PO,CDF,CCDF
        INTEGER*4 I
        SN=0.0
        PO=EXP (-TT)
        CDF=PO
        CCDF=1.0-CDF
C       WRITE(6,10) TT,SN,PO,CDF,CCDF
10      FORMAT(1X,'MEAN,SN,PO,CDF,CCDF ',5F9.5)
        IF(CDF.GE.0.005) CCDF=0.005
        IF(SS.LE.0.C) GO TO 26
        SN=SN+1.0
22      PO=PO*TT/SN
        CDF=CDF+PO
        CCDF=1.0-CDF
C       WRITE(6,11) TT,SN,PO,CDF,CCDF
11      FORMAT(1X,'MEAN,SN,PO,CDF,CCDF ','5F9.5)
        IF(CDF.GT.0.99999) GO TO 26
        IF(SN.GE.SS) GO TO 26
        SN=SN+1.0
        GO TO 22
```

```
TH201450
TH201460
TH201470
TH201480
TH201490
TH201500
TH201510
TH201520
TH201530
TH201540
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TH201560
TH201570
TH201580
TH201590
TH201600
TH201610
TH201620
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TH201680
TH201690
TH201700
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TH201720
TH201730
TH201740
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TH201810
TH201820
TH201830
TH201840
TH201850
TH201860
TH201870
TH201880
TH201890
TH201900
TH201910
TH201920
```

```
26    CONTINUE                                    TH201930
      RETURN                                      TH201940
      END                                         TH201950
```

COMPUTER PROGRAM FOR MARGINAL ANALYSIS

```
UMO000010
UMO000020
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UMO000390
UMO000400
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UMO000420
UMO000430
UMO000440
UMO000450
UMO000460
UMO000470
UMO000480
```

```fortran
C     ====  TEST FOR THE THREE INVENTORY MODELS ======
C
C     ================
C     ** OBJECTIVES **
C     ================
C
C          1. MINIMIZE UNITS SHORT MODEL
C          2. MINIMIZE TWUS          MODEL
C          3. MAXIMIZE PSEUDO AVAILABILITY MODEL
C
C     ** CONSTRAINT **
C          -- BUDGET ----
C
C     ** METHOD **
C          -- MARGINAL ANALYSIS TECHNIQE --
C
C     ** FUNCTIONS ***
C          1.PCCDF:  FIND C.C.D.F FOR POISSON DISTRIBUTION
C          2.PDEN ..: FIND PROB(X=N) POSSION DISTRIBUTION
C          3.TWS...:  FIND THE OBJECT FUNCTION FOR TWUS MODEL
C          4.UNITS..: FIND THE OBJECT FUNCTION
C          5.DELU...: FIND THE (Z(S-1)-Z(S))*E/C  FOR MODEL 1
C          6.DELTWS: FIND THE (Z(S-1)-Z(S))*E/C FOR MODEL 2
C          7.DELAV : FIND THE (Z(S)-Z(S-1))*E/C FOR MODEL 3
C
C     ** VARIABLE DEFINITION
C          LAM(I) ..: POISSON ANNUAL DEMAND
C          T(I) ...: LEAD TIME
C          C(I) ...: PRICE FOR AN ITEM
C          ESS(I) ..: ESSENTIALITY CODE
C          MTTR(I) : MEAN TIME TO REPAIR
C          MSRT(I) : MEAN SUPPLY RESPONSE TIME
C          AAV (I) : AVAILABILITY
C          US (II) : UNIT SHORT
C          S(I ) .: STOCK VECTOR
C          RATIO(I): Z(S)-Z(S+1)
C
      REAL LAM(100),T(100),C(100),ESS(100),MTR(100),MSRT(100),
     /AAV(100),US(100),S(100),RATIO(100),
     /B,BR,RR,OBJ1,OBJ2,OBT3,LTM,MTBF,X
      INTEGER N,I,K J,KK
C
C     INPUT DATA
C
      WRITE(6,9)
      N=10
```

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```
      DO 1 I=1,N                                                        UM000490
      READ(5,12) LAM(I),T(I),C(I),MTTR(I),ESS(I)                        UM000500
      WRITE(8,13) I,LAM(I),T(I),C(I),ESS(I),MTTR(I)                     UM000510
    1 CONTINUE                                                          UM000520
    9 FORMAT(1H1,10X,'    NO',5X,'LAMDA',5X,'LEAD T',5X,'PRICE ',5X,    UM000530
     /'ESS',5X,'MTTR '//)                                              UM000540
   12 FORMAT(5F10.4)                                                    UM000550
   13 FORMAT(10X,I5,5X,F5.1,5X,F5.1,5X,F6.1,5X,F5.1,5X,F10.4)           UM000560
      B=1170.                                                           UM000570
C                                                                       UM000580
C     INITIALIZE                                                        UM000590
C                                                                       UM000600
      DO 15 I=1,N                                                       UM000610
      S(I)=0                                                            UM000620
      RATIO(I)=DELU(LAM(I),T(I),1.0,C(I),ESS(I))                        UM000630
      RATIO(I)=DELTWS(LAM(I),T(I),1.0,C(I),ESS(I))                      UM000640
      RATIO(I)=DELAV(LAM(I),T(I),1.0,ESS(I),MTTR(I))                    UM000650
   15 CONTINUE                                                          UM000660
C                                                                       UM000670
C     ALLOCATE ITEMS WITH GIVEN BUDGET                                  UM000680
C                                                                       UM000690
      BR=B                                                              UM000700
   19 KK=0                                                              UM000710
      RR=-1.                                                            UM000720
      DO 16 K=1,N                                                       UM000730
      IF(C(K).GT.BR) GO TO 16                                           UM000740
      IF(RATIO(K).LE.RR) GO TO 17                                       UM000750
      RR=RATIO(K)                                                       UM000760
      KK=K                                                              UM000770
   17 CONTINUE                                                          UM000780
   16 CONTINUE                                                          UM000790
      IF(KK.EQ.0) GO TO 18                                              UM000800
C                                                                       UM000810
C     UPDATE RATIC(K),S(K),BR                                           UM000820
C                                                                       UM000830
      S(KK)=S(KK)+1.0                                                   UM000840
      RATIO(KK)=DELU(LAM(KK),T(KK),S(KK),C(KK),ESS(KK))                 UM000850
      RATIO(KK)=DELTWS(LAM(KK),T(KK),S(KK),C(KK),ESS(KK))               UM000860
      RATIO(KK)=DELAV(LAM(KK),S(KK),C(KK),ESS(KK),MTTR(KK))             UM000870
      BR=BR-C(KK)                                                       UM000880
      THETA=RR                                                          UM000890
      GO TO 19                                                          UM000900
C                                                                       UM000910
```

```
C     CALCULATE OBJECTIVE FUNTIONS WITH S VECTORS OBTAINED FORM THE
C        ABOVE STEP
      OBJ1=0.0
   18 OBJ2=0.0
      OBJ3=1.0
      SUMLTM=0.0
      DO 20 J=1,N
      LTM=LAM(J)*T(J)
      X=S(J)
      MTBF=1.0/LAM(J)
      SUMLTM=SUMLTM+LTM
      US(J)=UNITS(LAM(J),T(J),X)
      MSRT(J)=TMS(LAM(J),T(J),X)/LTM
      AAV(J)=MTBF/(MTBF+MTTR(J)+MSRT(J))
      OBJ1=OBJ1+ESS(J)*US(J)
      OBJ2=OBJ2+ESS(J)+AAV(J)
      OBJ3=OBJ3*MSRT(J)*LTM
   20 CONTINUE
      OBJ1=OBJ1/SUMLTM
      OBJ2=OBJ2/SUMLTM
C
C     OUTPUT FCR THIS TEST
C
C
C
      WRITE(6,25)
      DO 26 I=1,N
   26 WRITE(6,27) I,S(I),US(I),MSRT(I),AAV(I)
   25 FORMAT(///1X,' === ALLOCATION OF ITEMS =====',///
     /5X,I3,2X,' S(I) ',5X,' US(I)',5X,'MSRT(I)',5X,' AVIL ')
   27 FORMAT(5X,I3,2X,F6.1,5X,F9.4,3X,F9.4,5X,F9.5)
      WRITE(6,30) BR,THETA,OBJ1,OBJ2,OBJ3
   30 FORMAT(//10X,'BUDGET LIMIT =',F10.2/
     10X,'BUDGET LEFT =',F10.2/
     10X,'THETA =',F10.6/
     10X,'OBJECT VALUE OF UNITS  SHORT MODEL =',F10.6/
     10X,'OBJECT VALUE OF TMUS         MODEL =',F10.6/
     10X,'OBJECT VALUE OF AVAIL.       MODEL =',F10.6//)
      STOP
      END
C     POISSON COMPLMENTRY CUMULATIVE DISTRIBUTION FUNTION
      REAL FUNCTION PCCDF(AL,TT,SS)
      REAL AL,TT,SS,PG,CDF,SN,AA
```

UM000970
UM000980
UM000990
UM001000
UM001010
UM001020
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```
      AA=AL*TT
      SN=0.0
      PCCDF=1.C
      IF(SS.LE.0.0) GO TO 26
      PO=EXP(-AA)
      CDF=PO
      PCCDF=1.-CDF
      IF(SS.EQ.1.0) GO TO 26
      IF(CCF.GT.0.9999) PCCDF=0.0001
      IF((SS-1.0).EQ.0.0) GO TO 26
   22 SN=SN+1.C
      PO=PO*AA/SN
      CDF=CDF+FO
      PCCDF=1.C-CDF
      IF(PCCDF.LE.0.00001) GO TO 26
      IF(SN.GE.(SS-1.0)) GO TO 26
      IF(CCF.GT.0.99999) PCCDF=0.0001
      SN=SN+1.0
      GO TO 22
   26 CONTINUE
      RETURN
      END
C
C POISSON MASS FUNTION
C
      REAL FUNCTICN PDEN (AL,TT,SS)
      REAL AL,TT,SS,AA
      SN=1.0
      AA=AL*TT
      PDEN=EXP(-AA)
      IF(SS.LE.0.0) GO TO 27
   28 PDEN=PDEN*AA/SN
      IF(SS.EQ.SN) GO TO 27
      SN=SN+1.0
      GO TO 28
   27 CONTINUE
      RETURN
      END
C
C TIME WAITED UNITS SHORT OBJECTIVE FUNTICN
C
      REAL FUNCTICN IMS(AL,TT,S1)
      REAL AL,TT,S1,Z1,Z2,Z3,AA
      AA=AL*TT
      Z1=AA*PCCDF(AL,TT,S1)
      Z2=2.0*S1*PCCDF(AL,TT,S1+1.0)
      Z3=S1*(S1+1.0)*PCCDF(AL,TT,S1+2.0)/AA
      TWS=0.5*TT*(Z1-Z2+Z3)
```

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```
      RETURN
      END
C
C     CALCULATE THE OBJECTIVE FUNTION FOR UNIT SHORT MODEL
C
      REAL FUNCTICN UNITS(AL,TT,S2)
      REAL AL,TT,S2
      UNITS=AL*TT*PCCDF(AL,TT,S2-1.0)-S2*PCCDF(AL,TT,S2)
      UNITS=AL*TT*PDEN(AL,TT,S2)+(AL*TT-S2)*PCCDF(AL,TT,S2+1.)
      RETURN
      END
C
C     THIS FUNCTION 'DELU' CALCULATES (E/C(Z(S-1)-Z(S)) FOR UNITS
C     SHORT MODEL
C
      REAL FUNCTICN DELU(AL,TT,S2,CC,E)
      REAL AL,S2,TT,CC,E
      DELU=PCCDF(AL,TT,S2)
      DEL1=UNITS(AL,TT,S2-1.0)
      DEL2=UNITS(AL,TT,S2)
      DELU=DEL1-DEL2
      DELU=(E/CC)*DELU
      RETURN
      END
C
C     THIS FUNTION COMPUTES  THE MARGINAL  EFFECTAS   INCREASE ONE MORE
C     ITEMS FOR TWCS MODEL CASE
C
      REAL FUNCTICN DELTWS(AL,TT,SS,CC,E)
      REAL AL,TT,SS,CC,E,DEL1,DEL2,DEL3
      DEL1=TWS(AL,TT,SS)
      DEL2=TWS(AL,TT,SS-1.0)
      DELTWS=(E/CC)*(DEL2-DEL1)
      DEL3=(TT-SS/AL)*PCCDF(AL,TT,SS)-(SS/AL)*PDEN(AL,TT,SS)
      DELTWS=(E/CC)*DEL3
      RETURN
      END
C
C     THIS FUNCTION COMPUTE THE MARGINAL EFFECT  AS INCREASE ONE MORE ITE
C     ( PSEUDC AVAILABILITY MODEL CASE)
C
      REAL FUNCTICN DELAV(AL,TT,S4,CC,E,MTR)
      REAL AL,TT,CC,E,MTR,S4,AL,A2,MST1,MST2,MTF
      MST1=TWS(AL,TT,S4)/(AL*TT)
      MST2=TWS(AL,TT,S4-1.0)/(AL*TT)
      MTF=1.0/AL
```

UM001930
UM001940
UM001950
UM001960
UM001970
UM001980
UM001990
UM002000
UM002010
UM002020
UM002030
UM002040
UM002050
UM002060
UM002070
UM002080
UM002090
UM002100
UM002110
UM002120
UM002130
UM002140
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UM002200
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UM002370
UM002380
UM002390
UM002400

```
A1=MTF/(MTF+MTR+MST1)                    UM002410
A2=MTF/((MTF+MTR+MST2)                   UM002420
DELAV=(ALOG(A1)-ALOG(A2))*E/CC           UM002430
RETURN                                   UM002440
END                                      UM002450
```

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